# Elaborate: Volumes of Vessels

## Purpose:

Solve a problem using technology. Compare strengths and weaknesses of different technologies. Generate a list of attributes to guide judicious use of technology.

## Descriptor:

Pose a problem to the participants. The problem uses bottles, vases, cylinders, or prisms each with a constant cross section to generate a function that shows the relationship between the volume of the water in a vessel and the height of the water. Ask participants to estimate the surface area of the water without measuring. They may measure and calculate the surface area of the water after gathering all of their data. Participants may use any of the technologies presented during the professional development. Participants will then generate a list of attributes to guide judicious use of technology.

## Duration:

2 hours

## TEKS:

A.1(A) Describe independent and dependent quantities in functional relationships;

A.1(B) Gather and record data and use data sets to determine functional relationships between quantities;

A.1(D) Represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

A.1(E) Interpret and make decisions, predictions, and critical judgments from functional relationships.

A.2(B) Identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

A.2(C) Interpret situations in terms of given graphs or creates situations that fit given graphs; and

A.2(D) Collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

A.5(A) Determine whether or not given situations can be represented by linear functions.

A.6(B) Interpret meaning of slope and intercepts in situations using data, symbolic representations or graphs.

8.8(B) Connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects.

## TAKS Objectives:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 3: Linear Functions
- Objective 10: Mathematical Processes and Mathematical Tools

## Technology:

- Graphing Calculator
- Spreadsheet
- TI-InterActive!
- Graph linking capability, such as TI-Connect or Casio Program-Link
- Word processing technology
- Presentation technology (e.g., Power Point)

## Materials:

**Advanced Preparation:** collect vessels each with a constant cross section, preferably with a volume less than 700 ml and a height less than 20 cm, copy activity sheet **Volumes of Vessels** for participants, transparencies

**Presenter Materials:** overhead graphing calculator, projection device for computer

**Per group:** 3 vessels, water, cups, rulers, graduated cylinders, titration pipettes, food coloring, tape, computer, tray with sides to collect any water spills, paper towels, funnel (optional)

**Per participant:** copy of the activity sheet **Volumes of Vessels,** graphing calculator, sentence strips in three different colors

## Leader Notes:

Prior to this investigation, collect vessels with varying dimensions, but each vessel should have a constant cross section. The vessels will primarily be rectangular prisms and cylinders. If you can find vessels shaped like triangular prisms, hexagonal prisms, or oblique prisms, they are even more interesting. Each group of two or three participants should have 3 vessels to use for data collection. In order to accurately measure the height of the water, participants may wish to place a few drops of food coloring into the water. Participants will use a ruler to measure the height of the water in the vessel. Participants will pour water into each vessel at various heights. You may want participants to do this data collection in a tray with sides lined with paper towels so that water does not spill onto the tables. They will measure the volume by pouring the water at each height into a graduated cylinder. Use the metric system for these measurements remembering that one milliliter is equal to one cubic centimeter. Be aware that some groups might measure the height of the water from the top of the table instead of the actual height of the water in the vessel. Some vessels will have a thick bottom. The thick bottom will cause some groups to get a *y*-intercept that is not zero. If participants measure from the bottom of the water, they should get a *y*-intercept of zero.

Each group should use the technologies presented during the professional development to justify and present their solution to the problem. If participants need additional technology help direct them to the Technology Tutorial for this activity. Give all groups an opportunity to collect data, answer the questions, and prepare a presentation of their solutions. If jump drives are available, have each group save their presentation, then display the presentations on the presenter computer. If jump drives are not available, have all of the groups participate in a "Gallery Walk." Participants will be able to see how other groups used technology and to ask each other questions about their use of technology. Lastly, participants will generate a list of attributes to guide judicious use of technology.

## Posing the Problem

*Use the transparency **Volumes of Vessels** to pose the problem for the Elaborate phase of this professional development session.*

# Volumes of Vessels

- Three vessels with different heights, shapes, and volumes are in your collection.
- You will fill each vessel with four different heights of water.
- After you fill each vessel to a height, measure the height of the water, and then measure the volume of water in the vessel by pouring it into a graduated cylinder.
- Fill in the three tables with your collected data.

*Sample data:*

| **Description of Vessel #1** | *Tall, skinny rectangular prism* |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| *2* | *30* |
| *6* | *85* |
| *9* | *130* |
| *12* | *170* |

| **Description of Vessel #2** | *Short, wide rectangular prism* |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| *2* | *115* |
| *6* | *340* |
| *9* | *505* |
| *12* | *675* |

| **Description of Vessel #3** | *Tall, skinny cylinder* |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| *2* | *20* |
| *6* | *60* |
| *9* | *90* |
| *12* | *115* |

- Does a relationship exist between the volume of water in a vessel and the height of the water?
- What is the relationship?
- Use technology to present and justify your answer.

The **surface area of the water** is a measure of the number of square units necessary to cover the surface of the water.

Your presentation should include
- A scatterplot
- A description of a reasonable domain and range
- A trend line
- Approximate surface area of the water for each vessel
- Justification of solutions
- Measured or calculated surface area of the water for each vessel
- An example of a problem you might ask students after they have completed this investigation

*Distribute the **Volumes of Vessels Activity page** to each participant. If participants are struggling prompt each group to discuss the following questions as they generate a solution.*

## Part 1: Understanding the Problem (5-10 min)

1. **What data are you gathering? What data collection tools are available?**
   *The data collected includes the height of the water in a vessel and the corresponding volume. Tools available include rulers, graduated cylinders, titration pipettes, vessels, and water.*

2. **How might you organize your data? How might a scatterplot be useful to answer this question?**
   *Sample responses might include organizing the data in a scatterplot. By inspecting the scatterplot, we might be able to generate a function rule for each vessel.*

3. **What are the independent and dependent variables?**
   *Height is the independent variable, and volume is the dependent variable in this situation. We control the height and measure the volume.*

4. **What are a reasonable domain and range for this situation?**
   *The domain is different for each of the vessels depending on the height of each vessel. The volume, or range, depends on the capacity of each vessel.*

5. **Should this set of data be considered discrete or continuous? Why?**
   *The sets of data are continuous over the reasonable domain. You can fill each vessel to any desired height.*

## Part 2: Making a Plan (5-10 min)

1. **How might you use a graphing calculator to organize, represent, and analyze this data?**
   *A sample response might be to enter the data into the lists, use the lists to generate a trend line, and test a function rule that models the data.*

2. **How might you use spreadsheet software to organize, represent, and analyze this data?**
   *A sample response might be to enter the data into the lists and use the lists to generate a line of best fit.*

3. **How might you create a summary document that explains and justifies your answer to the question?**
   *Sample responses might include creating a Word document using screen shots or using a spreadsheet technology to generate a scatterplot and line of best fit. For some participants generating a PowerPoint demonstration would be a summary document.*

## Part 3: Carrying Out the Plan and Answering the Question (50 min)

1. **Does each vessel have a constant rate of change?**
   *Yes, vessels with a constant surface area will have a constant rate of change.*

2. **Find appropriate function rules to model your data. Test your rules.**
   *Function rules will vary depending on the data collected. Function rules modeling the sample data are $y = 10x$, $y = 14x$, and $y = 56x$.*

3. **How are the function rules for the vessels the same? How are they different?**
   *If participants measured the actual height of the water, the y-intercepts should all be approximately zero. If they included the height of the base, the y-intercept will be larger than zero and approximate the thickness of the base. The slopes will most likely be different. The vessels with larger surface areas have greater rates of change.*

4. **Explain why one function rule might be more or less steep than another.**
   *The vessels with larger surface areas will have a steeper function rule.*

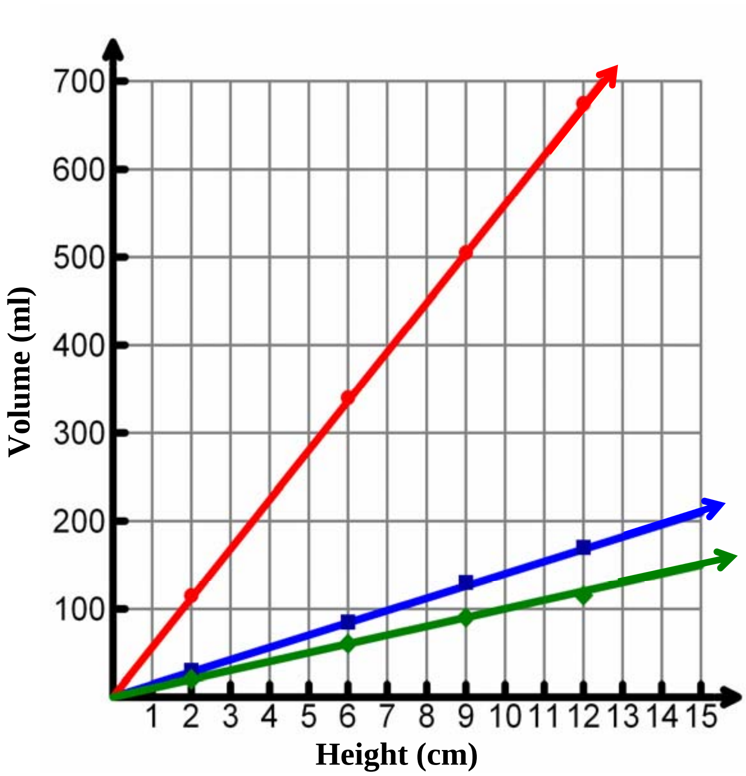
5. **After answering all of the preceding questions, measure and calculate to find the surface area of the water of each vessel.**
   *The surface area of each vessel which is approximated by the slope of the linear model should be approximately equal to the calculated surface area of the water.*

6. **Find the approximate surface area of the water in each vessel using only your function rules and the data you collected in the table. How is your answer related to the formula for volume of a prism or cylinder on the TAKS Mathematics Chart? Is your answer approximately the same as your answer to the previous question? Use technology to present and justify your answer.**
   *Sample solutions are presented on the following pages.*

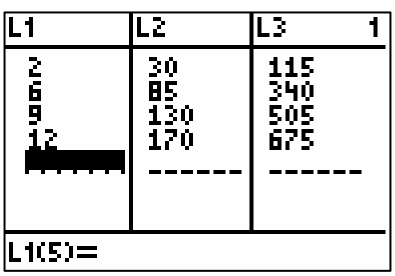
**Facilitation Questions**

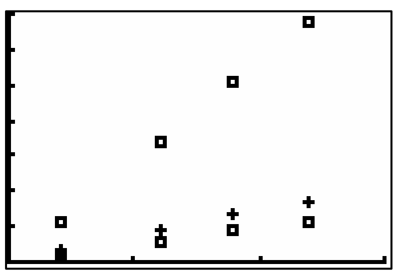
- What type of function does this appear to be?
- What did you observe?
- Is it possible to relate the generated linear function to a formula on the TAKS Mathematics Chart?
- How does technology assist us in interpreting the data?
- How does technology assist us in communicating the data?

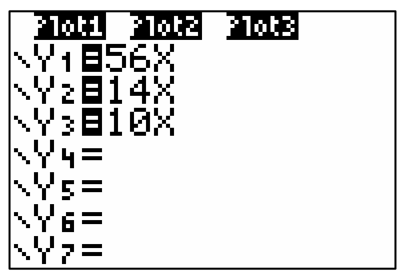
*Sample answer:*
*If participants require help with the technology refer them to the Technology Tutorial for Volumes of Vessels. Using TI-Interactive software, participants could graph the linear functions and the scatterplot for the three vessels on the same set of axes.*

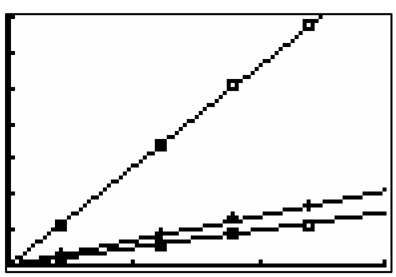



*Using the graphing calculator and a computer, participants might use TI-Connect to place screenshots in a Word document and then justify their solution.*

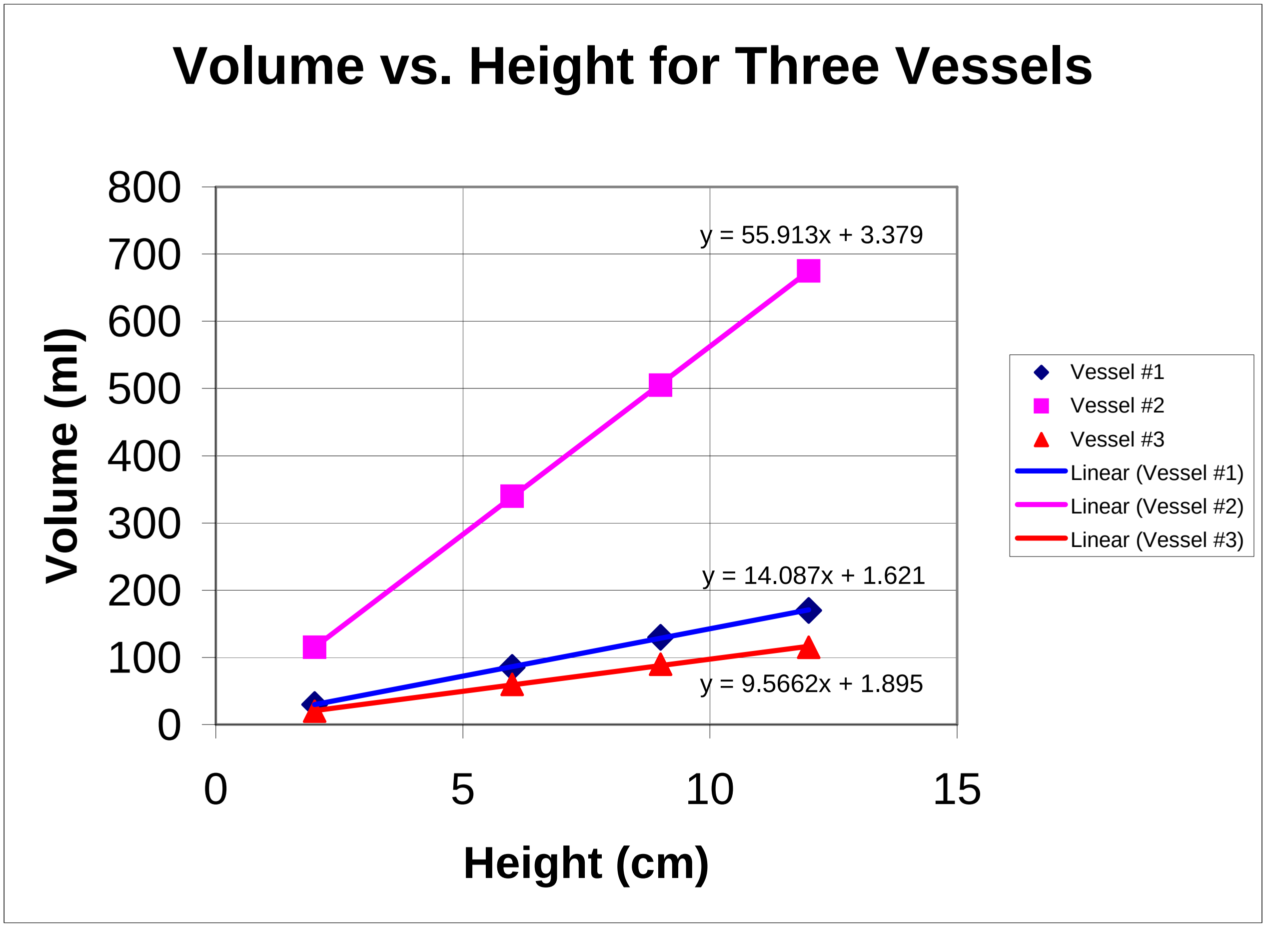




*Using a spreadsheet, participants may generate a graph using the collected values.*




*Some participants may wish to use Microsoft PowerPoint to present their solution. Have portable storage devices available for their presentations. Encourage participants to verbally describe their solution in addition to a graphical solution.*

*All participants should be able to communicate the "aha" moment when they notice that the slope of the linear function is the surface area of the water in each vessel. Since $V = Bh$ for any container with a constant rate of change, the linear function $f(x) = mx$ is a model for the volume vs. height data. The surface area of the water is equal to the "area of the base." The slope of the function is the "area of the base" portion of the formula. The volume is the sum of the "slices" of cross sections stacked to a given height. If you used any oblique prisms, the volume is equal to the surface area of the water multiplied by the perpendicular height.*

## Part 4: Evaluating the Answer and the Plan

1. **Is your solution reasonable? Why?**
   *Sample response: Yes, our solution is reasonable. The slope of the linear function is approximately equal to the calculated surface area of the water. Since V = Bh is the formula for the volume of a prism or cylinder, the slope of the linear function should be equal to the area of the base of the prism or cylinder.*

2. **Did you alter your plan while carrying it out? Why?**
   *Sample response: Yes, when we started measuring the height of the water, we measured the height from the top of the table rather than the bottom of the water. We decided that we wanted to measure only the height of the water in the vessel.*

3. **If you do this activity again, which technology tool(s) will you select to carry out your plan? Why?**
   *Sample response: We would use TI InterActive! because we felt that we were able to test our function rules and label the axes easily. We also like the graph from TI InterActive! better.*

## Part 5: Extending the Question

1. **What is the function rule for volume of water vs. height of the water for a vessel with a surface area of 5.2 square centimeters?**
   $f(x) = 5.2x$

2. **Give an example of a problem you might assign to your students to solve after they have completed this investigation.**

   *Sample Problem #1: What would the height of the water in each vessel need to be if each vessel were filled with 215 ml of water?*

   *Sample Problem #2: If each vessel were filled to a height of 5.5 cm, how much water would be in each vessel?*

   *Sample Problem #3: What would the graph of volume vs. height look like for a vessel when the surface area of the water is 45 $cm^2$? What is the function rule in this situation?*

3. **How might answering this question provide an opportunity for students to connect the formula for the volume of prisms and cylinders to the function rule that models each situation?**
   *Sample response: Many students do not connect geometry and algebra. This is an opportunity for them to see how a function rule can model the volume of a prism or cylinder and to observe how the rate of change, or slope, is related to the area of the base.*

*At this point allow groups to present their solution by using the jump drives. Prompt participants to disconnect the jump drive using the "Safely Remove Hardware" tool on the bottom menu bar and move the jump drive to the presenter computer. Display the documents for comparison purposes. If jump drives are not available, suggest that each group do a "Gallery Walk" and investigate the solutions of other groups.*

## Part 6: Technology Reflection (30 min)

1. *Upon completion of the **Volumes of Vessels** activity, prompt participants to work in pairs to brainstorm the role(s) technology played in the process of gathering, representing, and analyzing data. How does this compare to using only paper-and-pencil for this activity?*

2. *Repost the Venn diagram summaries from the engage phase.*

3. *Prompt participants to collect the "green sheets" from each Explore/Explain phase, the summaries about the intentional use of data that followed each Explore/Explain phase.*

4. *Display the **Transparency: Teaching Strategies** and prompt participants to reflect on the following question, "How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect the following teaching strategies for developing judicious users of technology?"*

**Facilitation Questions**

- How have the experiences in this professional development promoted careful decision-making about the appropriate use of technology?
  *Participant responses might include:*
  *During the professional development, participants decided what data they wanted to collect in **Motion**.*
  *In the Tree Ring Data Exploration lesson participants made decisions about which technology was appropriate to use.*
- How was technology integrated into the teaching and the learning of the TEKS?
  *Participant responses might include:*
  *We used technology as a tool to collect and explore data.*
  *We used technology as a tool to explore algebraic relationships.*
- When did you promote technology use? Why?
  *Participant responses might include:*
  *We promoted technology use when we collected data using the CBR in the Rainforest Canopy Tour.*
  *We promoted technology use in Volumes of Vessels to connect geometric and algebraic ideas concerning the volume and height of prisms and cylinders.*
- Was technology use restricted? If so, why?
  *Participant responses might include:*
  *Even though there were moments where technology was not used, its use was never restricted during the professional development. However, in the classroom it is appropriate to restrict technology use occasionally.*
- How did the technology support anticipatory, or "what if…", thinking about "algebraic insight"?
  *Participant responses might include:*
  *Technology encouraged us to predict what might happen, then check our prediction using a function rule.*
  *"What if" the area of the base changes, what happens to the graph?*
  *"What if" the tree is older, what happens to the radius?*

5. *Prompt the participants to respond to the following statement and question: "A successful teacher is one who uses technology judiciously. What does this ideal teacher look like and sound like?" Display **Transparency 1: Looks Like – Sounds Like**. Record the participant responses on sentence strips. Post the sentence strips randomly so that they are visible to the entire group. Use participants as scribes to facilitate the recording process.*

6. *Prompt the participants to respond to the following statement and question: "A successful student is one who uses technology judiciously. What does this ideal student look like and sound like?" as described on **Transparency 2: Looks Like – Sounds Like**. Record the participant responses on sentence strips. Post the sentence strips randomly so that they are visible to the entire group. Use participants as scribes to facilitate the recording process.*

7. *Direct the participants to work in small groups to brainstorm categories for classifying the "looks like" and "sounds like" responses.*

**Facilitation Questions**

- Do any of these responses require the teacher or the student to make decisions about technology use? Is this important? Should we add some responses?
  *Answers may vary.*
- Do any of these responses reflect decision making about how to best integrate technology? Is this important? Should we add some responses?
  *Answers may vary.*
- Do any of these responses reflect decision making about when to use or when not to use technology? Is this important? Should we add some responses?
  *Answers may vary.*
- Do any of these responses reflect the need for thinking about how the technology provides "algebraic insight" or "geometric insight"? Is this important? Should we add some responses?
  *Answers may vary.*

8. *As a whole group, debrief the categories created by small groups. Reorganize the sentence strips into broad categories. As a whole group, create titles for each of these categories. Record each title on a separate sheet of chart paper. Post the chart paper and reorganize the related sentence strips as shown below. Enlist participants to help with this process.*

Sample Category: Student Choice

The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide.

The student chooses to use a scatterplot instead of a table to represent her data.

9. *Prompt the participants to consider adding additional statements to any of the categories listed above that are not already posted. Reorganize "looks like, sounds like" sentence strips as needed.*

10. *Distribute to each group sentence strips that are a different color than the previously used sentence strips. Prompt each group to generate two classroom suggestions for each* ***category****. Examples may include, "The teacher should ask, 'Should we use the spreadsheet to make our prediction or verify our prediction? Why?'," "Students monitor their own use and misuse of technology," "Include examples that require technology use," "Do not allow students to use technology until after predictions are made and justified."*

*11. Prompt participants to post their sentence strips as shown below.*

| |
|---|
| Sample Category: Student |

| |
|---|
| The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide. |

| |
|---|
| The teacher posts a transparency that says, “Which tool will you use? Go there!” |

*12. Ask the participants to summarize any trends or patterns observed in the classroom suggestions.*

*13. Read the statement by Ball and Stacey as a closing thought to this phase of the professional development.*

**Facilitation Question**

- What is the value of this statement?
  *Answers may vary. It is encouraging to read that technology use is teachable. It makes me consider how I might better meet the needs of the student who doesn’t struggle with the math yet struggles with the technology.*

# Transparency – Volumes of Vessels

- Three vessels with different heights, shapes, and volumes are in your collection.
- You will fill each vessel with four different heights of water.
- After you fill each vessel to a height, measure the height of the water, then measure the volume of water in the vessel by pouring it into a graduated cylinder.
- Fill in the three tables with your collected data.

| **Description of Vessel #1** | |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| | |
| | |
| | |
| | |

| **Description of Vessel #2** | |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| | |
| | |
| | |
| | |

| **Description of Vessel #3** | |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| | |
| | |
| | |
| | |

- Does a relationship exist between the volume of water in a vessel and the height of the water?
- What is the relationship?
- Use technology to present and justify your answer.

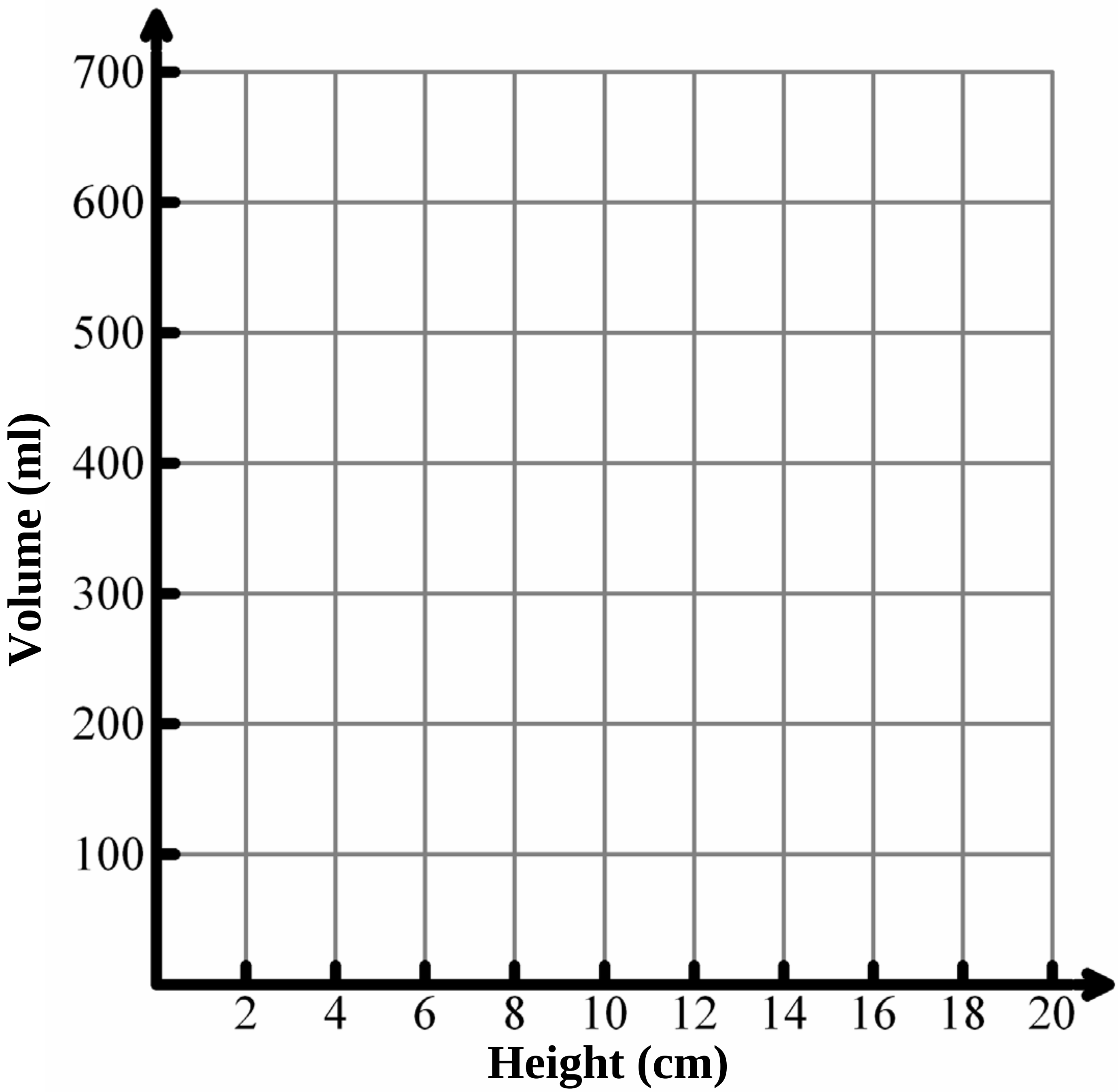
  The **surface area of the water** is a measure of the number of square units necessary to cover just the surface of the water.

Your presentation should include:

- A scatterplot
- A description of a reasonable domain and range
- A trend line
- Approximate surface area of the water for each vessel
- Justification of solutions
- Measured or calculated surface area of the water for each vessel
- An example of a problem you might ask students after they have completed this investigation

# Transparency: Volumes of Vessels

## Volume vs. Height



# Transparency – Teaching Strategies

"How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect the following four teaching strategies for developing judicious users of technology?"

Judicious users of technology:

a. Promote careful decision-making about the appropriate use of technology.

b. Integrate technology whenever relevant to the mathematical learning goals.

c. Promote and restrict the use of technology when appropriate for promoting mathematical learning

d. Promote anticipatory thinking about "algebraic insight" or "geometric insight."

# Transparency 1: Looks Like – Sounds Like

A successful teacher is one who uses technology judiciously.

What does this ideal teacher look like and sound like in this activity?

| Looks like… | Sounds like… |
| --- | --- |
| | |

# Transparency 2: Looks Like – Sounds Like

A successful student is one who uses technology judiciously.

What does this ideal student look like and sound like during the completion of this activity?

| Looks like… | Sounds like… |
| --- | --- |
| | |

# Transparency: Student Research

Research by Pierce (2002) indicates that some students are always judicious users and others persist with passive or random, unthinking use. However, she found that a large, middle group can be helped to learn to work judiciously.

Ball & Stacey, 2005, p. 5

Ball, L., & Stacey, K. (2005). Teaching strategies for developing judicious technology use. In Masalski, W. J., & Elliott, P. C. (Eds.), *Technology-supported mathematics learning environments, sixty-seventh yearbook*, pp. 3-16. Reston, VA: National Council of Teachers of Mathematics.

# Activity Master: Volumes of Vessels

- Three vessels with different heights, shapes, and volumes are in your collection.
- You will fill each vessel with four different heights of water.
- After you fill each vessel to the chosen height, measure the height of the water, and then measure the volume of water in the vessel by pouring it into a graduated cylinder.
- Fill in the three tables with your collected data.

| **Description of Vessel #1** | |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| | |
| | |
| | |
| | |

| **Description of Vessel #2** | |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| | |
| | |
| | |
| | |

| **Description of Vessel #3** | |
|---|---|
| **Height (cm)** | **Volume (ml)** |
| | |
| | |
| | |
| | |

- Does a relationship exist between the volume of water in a vessel and the height of the water?
- What is the relationship?
- Use technology to present and justify your answer.

The **surface area of the water** is a measure of the number of square units necessary to cover the surface of the water.

Your presentation should include:

- A scatterplot
- A description of a reasonable domain and range
- A trend line
- Approximate surface area of the water for each vessel
- Justification of solution
- Measured or calculated surface area of the water for each vessel
- An example of a problem you might ask students after they have completed this investigation

## Part 1: Understanding the Problem

1. What data are you gathering? What data collection tools are available?

2. How might you organize your data? How might a scatterplot be useful to answer this question?

3. What are the independent and dependent variables?

4. What are a reasonable domain and range for this situation?

5. Should this set of data be considered discrete or continuous? Why?

## Part 2: Making a Plan

1. How might you use a graphing calculator to organize, represent, and analyze this data?

2. How might you use spreadsheet software to organize, represent, and analyze this data?

3. How might you create a summary document that explains and justifies your answer to the question?

## Part 3: Carrying Out the Plan and Answering the Question

1. Does each vessel have a constant rate of change?

2. Find appropriate function rules to model your data. Test your rules.

3. How are the function rules for the vessels the same? How are they different?

4. Explain why one function rule might be more or less steep than another.

5. After answering all of the preceding questions, measure and calculate to find the surface area of the water.

6. Find the approximate surface area of the water in each vessel using only your function rules and the data you collected in the table. How is your answer related to the formula for volume of a prism or cylinder on the TAKS Mathematics Chart? Is your answer approximately the same as your answer to the previous question? Use technology to present and justify your answer.

## Part 4: Evaluating the Answer and the Plan

1. Is your solution reasonable? Why?

2. Did you alter your plan while carrying it out? Why?

3. If you do this activity again, which technology tool(s) will you select to carry out your plan? Why?

## Part 5: Extending the Question

1. What is the function rule for volume of water vs. height of the water for a vessel with a surface area of 5.2 square centimeters?

2. Give an example of a problem you might assign to your students to solve after they have completed this investigation.

3. How might answering this question provide an opportunity for students to connect the formula for the volume of prisms and cylinders to the function rules that model this situation?

# Volumes of Vessels

*Volume vs. Height*